The Inca Philosophy of 10

Viviana R. Moscovich

Abstract
In his Gramática y arte nueva de la lengua general o lengua del Inca, Gonçalez Holguín ([1607] 1842:217) describes the unique Quechua arithmetical system and presents the names of the numbers in Quechua in a decisively decimal outline, beginning in '1' and finishing in the 'infinite'—that is, a list of positive integers (Z+) that does not include 'zero.'
In his list, each numeral in Quechua appears with its translation into Spanish and a representation in western Indo-Arabic numerals. However, even though the systems would be seemingly identical (i.e., decimal), the representation of Quechua numerals in Western ones reveals, in fact, an essential cultural discrepancy, as the graphic representations of the Western numerals, from 0 to 9, and their combinations used to construct numbers higher than 10, did not exist in Inca culture, and their decimal system, as will be shown below, was based on utterly different concepts, structures and visualization means.
Among these concepts, the 'count by 10' and powers of ten, building on other basic concepts (such as hanan-hurin, for example), turns out to be an inherent element of Inca thought applied by these to structure and manage the Tawantinsuyu's territory and society.

Keywords: Inca, decimal system, khipu (Quipu), accounting, Quechua, khipu kamayuq (quipocamayoc), administration, governors, philosophy, province, Cuzco (Cusco), tambo (tampu), mathematics, Tawantinsuyu.

Resumen
LA FILOSOFÍA INCA DEL 10
En su Gramática y arte nueva de la lengua general o lengua del Inca, Gonçalez Holguín ([1607] 1842:217) describe el singular sistema aritmético quechua y presenta los nombres de los números en quechua en un esquema decididamente decimal, comenzando en el ‘1’ y terminando en el ‘infi nito’—es decir, una lista de enteros positivos (Z+) que no incluye el ‘0’.
En su lista, cada numeral en quechua aparece con su traducción al español y una representación en números indo-arábigos occidentales. Sin embargo, aunque los sistemas aparentarían ser idénticos (es decir, decimales), la representación de los numerales quechuas en numerales occidentales revela, de hecho, una discrepancia cultural esencial, ya que las representaciones gráficas de los numerales occidentales, del 0 al 9, y sus combinaciones utilizadas para construir números superiores al 10, no existían en la cultura Inca, y su sistema decimal, como se mostrará a continuación, se basaba en conceptos, estructuras y medios de visualización completamente diferentes.
Entre estos conceptos, el ‘conteo de 10’ y potencias de diez, construido sobre otros conceptos básicos (como hanan-hurin, por ejemplo), resulta ser un elemento inherente del pensamiento Inca, aplicado por estos para estructurar y gestionar el territorio y la sociedad del Tawantinsuyu.

Viviana R. Moscovich • Institute of Archaeology, Hebrew University of Jerusalem
vivianaruthm@gmail.com; viviana.moscovich@mail.huji.ac.il
1 Tawantinsuyu is the original Inca name of the so-called ‘Inca Empire.’
2 Tawantinsuyu es el nombre Inca original del llamado ‘Imperio Inca.’
Palabras clave: Inca, sistema decimal, khipu (Quipu), contabilidad, Quechua, khipu kamayuq (quipocamayoc), administración, gobernadores, filosofía, provincia, Cuzco (Cusco), tambo (tampu), matemáticas, Tawantinsuyu.

1. The Inca Decimal System
1.1 The “Accountants”

The foremost notable trait of this system is its intrinsic structural link with its visualization devices and with the titles of the people specialized in their use, devices specifically made for the system, or even, eventually, emerging from it.

In his Quechua grammar from 1607, Gonçaléz Holguín ascertained ([1607] 1842:217–218): “... before there were great quipocamayos, that are accountants, and they have names for as many numbers and accounts as we have in Castilian ...”  

These “accountants,” officials of the Tawantinsuyu, played a crucial role in its administration. They are also present in other chronicles of the 16th and 17th centuries written by both Spaniards and locals, such as that of Guaman Poma de Ayala (1615), of local origin, who described these “accountants and treasurers” of the Tawantinsuyu, the Historia natural y moral de las Indias, written by Father José de Acosta in 1590, or the Comentarios Reales de los Incas, written by Inca Garcilaso de la Vega in 1609. Noteworthy, in every written source, these accountants are the only individuals depicted or described using these numbers and their visualization systems. No other Inca/Tawantinsuyu officials are directly related to numbers, accounts, or their visualization methods.

Before discussing the central theme of this article, it is necessary to provide a very brief description of these “accountants” and their role in the Inca/Tawantinsuyu administration.

The said “accountants” were, in fact, administrators, officers named by the Incas in Cuzco and placed as administrators throughout the Tawantinsuyu. These officers were called khipu kamayuq for their ability to create and read khipus and annotate data on their cords. This generic title encompassed an extensive list of officers who specialized in khipu—from people responsible for the imperial warehouses to local accountants in villages, local administrators, and governors of provinces. According to my previous research, these officials originated in two different populations (Moscovich 2008a, 2016a, 2016b, 2016c): 1. Inca nobles from Cuzco and 2. Local nobles—kings and princes of conquered areas, called qhapaq apu or apu. These local nobles were sent to Main Cuzco, where, according to the written sources, 4 they studied for four years in the School of Cuzco—created by the Sapa Inca and run by an amawta, the so-called “wise men” of the Tawantinsuyu—the art of governing, of

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3 All the translations of written sources into English were done by the author of this article.
4 These schools are described by several chroniclers, the more explicit one being Martín de Murúa ([1613] 1987:376–377), who details who studied there (“sons of the principals and of the ore-jones”), what was taught, and what happened to the graduates. See also, among others, Garcilaso de la Vega ([1609] 19995:238–239).
making *kipu*, Inca religion and ceremonies, Inca laws and punishments, and Inca history.

According to these same sources, the local nobles, at the end of their reeducation, now having become Inca *kipu kamayuq*, worked at the Inca’s court in Cuzco for some time (trial period) and, if successful, were then returned to their reigns of origin, serving a double purpose: to rule over their own subjects as the legitimate local rulers and serve as the Inca/Tawantinsuyu’s administrators and accountants.

However, the Inca princes and nobles, of pure Inca blood and related to the Sapa Inca, were the only ones who achieved the status of “Governors of Provinces” (*t’uqrikuy =* governor): “Inspectors” (*tukuy rikuq*), and “amawta.” It should be pointed out that even though some of these Inca-blood *kipu kamayuq* became *amawta*, not all did. In contrast, based on the information from the written sources, it can be ascertained that the contrary is true and that the *amawta* were all *kipu kamayuq*. On the other hand, a *qhapaq apu* could be granted a place in the Sapa Inca’s Council in Cuzco. In summary:

<table>
<thead>
<tr>
<th>Nobles of Inca Blood (= Orejones)</th>
<th>Nobles from Conquered Kingdoms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incas of Cuzco = Orejones (already in Cuzco)</td>
<td>“Nobles of the Tawantinsuyu” “Ancient Kings and their children” (<em>qhapaq apu</em> and <em>apu</em>) (Called to Cuzco)</td>
</tr>
<tr>
<td>↓ They study for 4 years administration, religion, <em>kipu</em>, and Inca laws</td>
<td>↓ They study for 4 years administration, religion, <em>kipu</em>, and Inca laws</td>
</tr>
<tr>
<td>↓ They work in Cuzco for the Sapa Inca (trial period)</td>
<td>↓ The local kings and princes work in Cuzco for the Sapa Inca (trial period)</td>
</tr>
<tr>
<td>↓ The best are named Governors, and some are sent to the provinces as Inca Province Governors, can serve as inspectors, can become <em>amawta</em></td>
<td>↓ The best are returned to their original kingdoms and retake their command as local rulers, serving also as Tawantinsuyu administrators under the Inca Governors; the <em>qhapaq apu</em> can also become part of the Sapa Inca’s Council in Cuzco.</td>
</tr>
</tbody>
</table>

### 1.2 The Accountants and their Calculating Devices

Concerning the calculating devices used by the *kipu kamayuq* (the so-called “accountants”), Cieza de León, in *El señorío de los Incas* ([1553] 1985:58) wrote that they used the *kipu* (knotted strings) both for recording expenses and things from the past by means of a numbering system going from 1 to 10, 10 to 100 and 100 to 1000.

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5 Or governors of other things, such as *qhapaq ñan t’uqrikuy =* Governor of the Royal Road (see Guaman Poma de Ayala 1615:354 [356]), https://poma.kb.dk/permalink/2006/poma/356/es/text/
Father José de Acosta ([1590] 1986:403), on the other hand, wrote that they used a “sort of khipu” which functioned with grains of corn.\(^6\)

Because a very complicated calculation ... to see how much is due amongst many, how much of contribution ... these Indians will take their grains and put one here, three there, eight I do not know where; they will transfer one grain from here, will exchange three from there, and indeed they get their account done with very much accuracy, without missing a mark ...?\(^7\)

Martín de Murúa ([1590] 1946:223) wrote that calculations were carried out using stones and knots, counting 1, 10, 100, 1000, 10000, and 100000.

The different functions of the two counting devices (i.e., the khipu and the Yupana) are clearly detailed in Inca García de la Vega and Guaman Poma de Ayala's chronicles. García de la Vega wrote ([1609] 1995:128):\(^8\)

That by knots found on a few threads of diverse colors they [the accountants] gave an account of all the tributes and contributions in the kingdom of the Inca, by charge or discharge ... They added, subtracted and multiplied through those knots. And to know what was due by each village they did their partitions with grains of corn and small stones, and in this way their account resulted accurate.

Guaman Poma de Ayala (1615:361 [363]) adds that these devices were, in fact, used by three types of accountants, who counted either in khipus or Yupanas using a decimal system but not in both.\(^9\), \(^10\)

1. **Royal Accountant of all this kingdom Condor Chaua son of apu - He was called-**

   **Tawantinsuyu runaquipoc yncap haciendan chasquicoc\(^11\)- major treasurer -** They say this principal had a great ability -

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\(^6\) Acosta’s description seems to correspond to the counting device named Yupana or counting table. See Mosovich 2006, 2007, 2016a, 2020 for further discussion of this device.

\(^7\) ... Pues verles otra suerte de quipos que usan granos de maíz, es cosa que encanta. Porque una cuenta muy embarazosa ... para ver a cómo les cabe entre tantos, tanto de contribución ... tomarán estos indios sus granos y pondrán uno aquí, tres aculá, ocho no se dónde; pasará un grano de aquí, trocarán tres de aculá, y en efecto ellos salen con su cuenta hecha puntualísimamente, sin errar un tilde ...

\(^8\) Que por nudos dados en unos hilos de diversos colores daban cuenta de todo lo que en el reino del Inca había de tributos y contribuciones, por cargo y descargo. Sumaban, restaban y multiplicaban por aquellos nudos. Y para saber lo que cabía a cada pueblo hacían las particiones con granos de maíz y piedrezuelas, de manera que les salía cierta su cuenta.

\(^9\) 1. Contador Mayor de todo este reino condorchaua hijo de apu –

   A este le llamaban Tawantinsuyu runaquipoc yncap haciendan chasquicoc-

   tezorero mayor [espacio pequeño]

   Dice[n] que este principal tenía [una] gran auilidad

   Para saber su abilidad el Ynca mando contar y numerar, ajustar con los indios de este reino –

   [ ] con la lana del cierbo. [:] taruga, emparejaba con la lana a los indios – [ ]...\(^1\)

\(^10\) Punctuation marks and bold texts by the author of this article. Brackets: [ ] = added to the original text.

\(^11\) **Tawantin Suyu runa khipu Inqap haciendan chasquikuq = the person who annotates on the khipu the subjects of Four Quarters, and receiver of the Inca’s incomes.**
To know his ability the Ynca ordered to count and number and level with the Indians of this kingdom
(with the wool of the cerf, taruka, he matched the Indians with the wool -)

2. Major accountant Hatun Huchaquipoc\textsuperscript{12} – Minor accountant Huchuy Huchaquipoc – they count in tables: they number one hundred thousand and ten thousand and one hundred and ten until they reach one...

It should be noted that the numbers Guaman Poma de Ayala gave are inverted in order (higher to lower) and that the number 10000 is missing. Also, it is important to note that he writes “they number” and not “they count.”

These chroniclers provide several important details concerning the arithmetical system of the Inca and the devices used for calculations and registering numbers:

1. The *kipus* were not used for dividing but only for the other three arithmetic operations: adding, subtracting, and multiplication.

2. Dividing, an operation that served to calculate what was due by/to each village or person, was done on the *yupana*, the counting table, which was also used for all the other three operations.

3. The *kipu kamayuq counted* from bottom to top, that is, from one to infinite, following a decimal system.

4. However, they *numbered* from top to bottom, using powers of ten: 100000, 10000, 1000, 100, and 10. However, from 10 to 1 there is a continuous count.

5. Registering data was done on the *kipus* using knots arranged in two distinct sections: charge and discharge, or *habet* and *debet*, income and expenses, the basic system used today by any accountant, but not by the Spanish at the time. Such a double-registry *kipu* was seemingly drawn by Guaman Poma de Ayala in his chronicle (1615:360 [362]) (Fig. 1):

![kipu](image)

Fig. 1. A habet–debet kipu—23 cords (left) and 32 cords (right).\textsuperscript{13}

\textsuperscript{12} Hatun Huchaquipoc – the one who annotates on the *kipu* big issues; Huchuy Huchaquipoc – the one who annotates on the *kipu* small issues.

\textsuperscript{13} Image by Julia Skidel-Rymar on the original by Guaman Poma de Ayala 1615: 360 [362].
1.3 The Inca Decimal System – its origins and its relation to the Quechua language

Several questions arise from these descriptions, the main one being whether the decimal system used by the Inca was specific to them, was their invention, or if they took an already known system and just developed it further.

The idea of the khipu as a recording device was not an Inca invention. However, the specific types of khipu used by the Inca and the Tawantinsuyu seemingly were. The most important collection of earlier khipus dates from the Middle Horizon and was seemingly created by the Wari culture. These khipu were carbon-dated to around 700 CE and studied by Urton. He states that:

*Middle Horizon khipus display no evidence of a foundation in decimal numerical organisation. Rather, ... many Middle Horizon examples bear single/overhand knots, and these are often arranged in closely clustered groups of five knots (there are many single, or lone, knots as well). There is a suggestion of an interest in values from one to five in the Middle Horizon examples, but no evidence for decimal recording.*

*On the other hand, the majority of Inka khipus were organised to a high degree around the recording of numerical values in decimal place-value tiered hierarchical arrangements.* (2014:217)

Furthermore, by analyzing the names of the 10 first numbers in each of the languages spoken by peoples in Wari and Inca areas: Quechua, Aymara, and Puquina, Urton concludes that Puquina and Quechua (both spoken by the Inca) “... Have independent lexemes for the numbers one to ten; both are decimal-based systems of numeration,” while in Aymara:

“there are independent lexemes for the numbers one to five ... Aymara appears to have originally had a quinary, or base five, system of numeration ... we could say that Puquina and Quechua numeration are consistent with, or accommodate, the decimal numeration principle evident in Inka khipus. On the other hand, if indeed the Middle Horizon khipus are Wari, and if Wari peoples spoke Aymara, then we could say that there is a coincidence in Wari cord technology of a base five terminological system and (perhaps) knotting technology.” (Urton 2014:217–218)

If the Inca did not adopt an earlier empire-like culture’s arithmetical system, what could have been the origin of the Inca decimal system? And how do Inca khipu structurally and ideologically reflect it?

In his 17th-century Quechua–Spanish vocabulary, Gonçalez Holguín ([1608] 1952:121) described a calculating system by “tens in heaps” (“diezes en montones”) used in the Tawantinsuyu’s area:

*Chunkachani.* To lay down by the tens.

*Chunkachini, o chunkachaqarinini:* To count by heaps of tens.

*Chunkakahsqaktam yayanchani.* To sum up the tens or the hundreds or the thousands into one sum and remove the heaps.
Yayanchasqantam wañuykuni. To sum up all the tens and the hundreds to a single number.\textsuperscript{14}

Radicati de Primeglio also reported, in 1990, a traditional way of counting, very similar to the one described by Gonçalez Holguín, also based on heaps of tens, still used in the region of Paucartambo in 1990 and described by Oscar Núñez del Prado in 1951:

*The account … is done by putting aside one potato from every ten units that are put in the sack, then immediately proceeding to the simplification by substituting every ten tens with other potatoes that indicate the hundreds, then marking every ten groups of one hundred with other potatoes designating the thousand thousands.* (1990:234)\textsuperscript{15}

A comparison of the texts of Gonçalez Holguín (1608) and Radicati de Primeglio (1990), describing a very similar or identical counting system, shows that this system works as follows:

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity of 1 ⇒</td>
<td>1 Potato</td>
</tr>
<tr>
<td>Quantity of 2–9 ⇒</td>
<td>2–9 Potatoes in a heap</td>
</tr>
<tr>
<td>Quantity of 10 ⇒</td>
<td>1 Potato = one unit that symbolizes 10</td>
</tr>
<tr>
<td>Quantity of 11–19 ⇒</td>
<td>1 Potato (=10) + 1–9 potatoes in a heap</td>
</tr>
<tr>
<td>Quantity of 20 ⇒</td>
<td>2 Potatoes = 2 units of 10 (2 times 10)</td>
</tr>
<tr>
<td>Quantities of 20–90 ⇒</td>
<td>2–9 Potatoes = 2–9 units of 10</td>
</tr>
<tr>
<td>Quantity of 91–99 ⇒</td>
<td>9 Potatoes = 9 units of 10 (=90) + 1–9 potatoes in a heap</td>
</tr>
<tr>
<td>Quantity of 100 (=10 times 10 or 10(^2)) ⇒</td>
<td>1 Potato = one unit that symbolizes 100 (10 times 10)</td>
</tr>
<tr>
<td>Quantity of 101–199 ⇒</td>
<td>1 Potato (=100) + 1–9 Potatoes (10–90) + 1–9 Potatoes (1–9)</td>
</tr>
</tbody>
</table>

Thus, according to this method, two, three, and up to nine potatoes are considered heaps, or intermediate numbers between the unit of 1 and the new ‘1’ symbolizing 10 and powers of ten (10\(^2\), 10\(^3\), 10\(^4\)). Fig. 2 shows this graphically.

Both methods transcribe the oral expressions of numbers (1 to 10, 11 to 20, and so forth) into a simplified visual representation, where 1 potato (or any quantity of 1) symbolically represents a unit of 10, 100, 1000, 10000, and so on, respectively.

\textsuperscript{14} Chuncachani. *Poner de diez en diez.*
*Chuncachini, o chuncacharcarini. Contar por diezes en montones*
*Chuncachascactam yayanchani. Sumar los diezes o cientos o miles en un tanto y quitar los montones.*
*Yayanchasqantam huañuykuni. Resumir todos los diezes y cientos a un número.*

\textsuperscript{15} La cuenta … se hace separando una papa por cada diez unidades que se echan al costal, en seguida se procede a la simplificación, sustituyendo por cada diez decenas otras tantas papas que indican las centenas y señalando luego, cada diez grupos de cien, con papas representativas de los mil millares.
Thus, in this system, these numbers, even when having proper names in the oral language, cease to be considered 10, 100, 1000, or 10000 units of potatoes to become ONE key unit of either 10, 100, 1000, or 10000 (or powers of 10: $10^1$, $10^2$, $10^3$, $10^4$, and so forth).

Also, in both methods, once we arrive at a quantity of 100 potatoes/a heap of 100, actually perceived as 10 times 10, one potato replaces the 9 potatoes that symbolized quantities/heaps of 10 potatoes before that. Thus, the potato symbolizing a unit of 100 includes the nine 10-potato units, which can be discarded. In addition, any quantity in between these decimal key units becomes a ‘heap,’ a transitional agglomerate filling the space and belonging to the decimal integer that preceded it:

- 10 potatoes/10 units in a heap = 1 unit of 10
- 11–19 potatoes/11–19 units in a heap = 1 unit of 10 + 1–9 = $1 \times 10 + 1–9$ that still belong to the single unit of 10
- 20 potatoes/20 units in a heap = 2 units of 10 = $2 \times 10$

The above means that any numbers between 11 and 19, in this case, are still counted on the basis of $1 \times 10 +$ something, either belonging to or being added to that decimal unit (10) and are certainly NOT considered 11–19 units as in Western counting. Numbers between 21 and 29 will belong or be added to the $2 \times 10$, the
decimal unit of 20. Thus, after reaching \(10^2\) (100), numbers cease to belong to the tens units to belong to 100, which is \(10 \times 10\).

This is very interesting since this concept concurs with the numerical expressions in Quechua, where the number 12, for example, is called \(\text{chunka iskayniyuq}\), that is 10+2 (10 with 2; 10 that has 2), the suffix ‘yuq’ serving as a possessive, but where 20 is called \(\text{iskay chunka}\), meaning ‘two ten,’ without any belonging, or possession suffix. This occurs from the unit of 10 up to 99.

After 100, having changed to the new power of 10 (\(10^2\)), all the numbers up to 199 will be counted using 100 as their base: 1 unit of 100 + a number added (Figure 3 shows examples of how numbers are annotated on a \(khipu\) cord):

\[
101 = \text{Pachak hukniyuq} = 100 \text{ with 1, 100 that has 1}
\]

\[
120 = \text{Pachaq iskay chunkayuq} = 100 \text{ with 20, 100 that has 20}.
\]

However, this ‘unit of 10’ concept goes even much deeper into Inca Quechua thought. In his vocabulary, González Holguín ([1608] 1952:121) brings the terms “\(\text{Chunkaymi, aylluym}\)”\(^{16}\), \(^{17}\) as synonyms, to which he gives the meaning of: “He is my relative, of my lineage.” Thus, the \(\text{ayllu}\), a basic-level Andean and Inca unit paral-

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\(^{16}\) Chuncaymi. From \(\text{chunca}\), which means ‘10’ = “My ten.” Aylluym means “my ayllu.”

\(^{17}\) Chuncaymi, o aylluym. Es mi pariente, de mi linaje (González Holguín [1608] 1952:121).
2. Implementation of the Decimal System

2.1. The khipu

In the counting system described above, by heaps of tens, a logistic problem has to be solved since to count, for example, 199 potatoes (or any other product), the person counting would have to use some hierarchical system (e.g., by place, or different organization) to differentiate between the potatoes representing 100, 90 (9 times 10, which could make 9 potatoes in a heap) and 9 (nine potatoes in a heap)—in this order, from the highest number to the lowest one—since once that person gets to 100, this number serves as the first number of the count, upon which the other numbers are added.

The Inca khipu, the tool of the official accountants of the Inca and the Tawantinsuyu, actually resolves this logistic problem. It reflects the Inca decimal counting system and the ‘numbering’ system described by Guaman Poma de Ayala (1615, see above), with the numbers marked on pendant cords as knots placed hierarchically from top to bottom representing powers of ten, from 10000 (10⁴) downwards. Also, it is important to note that the khipu differentiates between these powers of ten, as key numbers, the numbers 2–9, and even the single unit 1: The powers-of-ten knots (1₀, 1₀, 1₀, 1₁) are each represented by the so-called “simple knots;” the units 9–2, placed below the 10-knot, are represented by the so-called “long knots,” agglomerations shaped as a closed fist, somewhat difficult to recognize; and the knot designating ‘1’ resembles in shape the western number ‘8’ (Fig. 4).

The khipu cord behaves, thus, as a tangible transcription of the count by heaps of tens/potato count (Fig. 5), ‘numbering’ from top to bottom (in contrast to ‘counting,’ which, as explained above, is done from 1 to the infinite). If we arrange the potato count and the khipu cord horizontally, we will see the following:

Potato count (= ‘counting’):
1----2--9----10000000
Khipu cord (= ‘numbering’):
10000000000100000000--1000000--10---9--2----1

Thus, in the Inca khipu, one ‘simple knot’ represents a unit of 10000, 1000, 100, or 10 (powers of 10), parallel to the one potato/other product representing these units. This graphic representation as knots on the khipu cord solves the problem of differentiating between the different values when using the same product (in this case, potatoes) in higher numbers (as in the number 199, for example, as explained above).

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18 Versions of the texts in Section 2 (examining these topics in various contexts) have been published in Moscovich 2008b, 2016a, 2016b, and 2020 in English and Spanish.
Fig. 4. The Khipu Cord: Knot Types (Based on Locke 1923).

Fig. 5. The khipu knots as a tangible graphic ‘numbered’ transcription of the ‘potato and heaps of ten counting methods.’
Numbers 2–9, symbolized by an agglomeration-like knot, reflect amounts between the unit of 1 and the unit of 10, and it should be noted that differentiating between numbers 6 and 7, or 8 and 9, for example, can reveal a difficult task, while number 1 is always clearly differentiated by a special knot-type, and so are the powers of ten knots. Thus, it is not by mere chance that the Inca accountants chose these agglomeration-like knots (‘long knots’) to represent the numbers 2–9 since these numbers look like something additional or undefined situated between the unit of one and the first decimal unit (10’). 

Also, the fact that these ‘long knots’ somehow look like a closed fist is not a coincidence, as explained in the following section.

2.1.1 The Empty = ‘Zero’ Equation

The khipu has no specific knot designating ‘zero’ or ‘empty.’ Nor does this notion of empty = zero exist in Quechua. It is important to point out the complete absence of the idea of ‘empty’ or the number ‘0’ in the names of numbers in Quechua:

1100: waranka pachakniyuq (‘1’ unit of 1000 with ‘1’ unit of 100)
1010: waranka chunkayuq (‘1’ unit of 1000 and ‘1’ of 10).

In western eyes, the absence of a knot in the place of the 100 in the number 1010 will be perceived as a vacuum between the knot designating 1000 and the knot designating 10 below it. However, for the khipu kamayuq who built the khipu and used it, the same space would not be empty but full.

As explained above, arriving at a power of ten—both in potato counting or khipu numbering—denotes a step represented by a key number encompassing all the numbers/items counted before and which are part of it. Thus, the number 100 will include all the numbers 1–99 as an integral part of it, and 100 will represent a group of 100 related elements, like the ayllu, which comprises 10 related households, and the two hands that comprise the 10 fingers, all interrelated. A number, on its own, does not seem to exist in Inca thought and is always related to some element or produce.

This equivalence empty space = full (of something) corresponds to the findings of Salomon (2004:215) in the village of Tupicocha. He writes about cords without any knots that “Nery raised the matter of cords without knots without prompting. ‘A cord without a knot. A member of the community fulfilled all his duties as member of the ayllu . . .’”

Thus, it can be said that the possession relation between numbers in Quechua is fixed and determined by oral language. However, on the khipu cords, this relation is recognized through the spaces that we might see as empty but actually represent fullness. Therefore, there is no place for the empty = zero equation or the zero number in the Inca khipu, numbering, and accounting systems.

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19 A young man from Tupicocha who interpreted for Salomon the knots and cords of a khipu kept in his village.
2.2. The Hanan-Hurin and ‘10’ Philosophies

The Inca decimal system was inherent to Quechua language and Inca thought, reflected in Inca khipu and contrasting with the Aymara language and the Wari khipu (see Urton 2014 above, in Section 1.3).

Interestingly, a conceptual and structural inherent element of Inca culture explains why the Inca could have never had a quinary numerical system but only a decimal one: the Hanan–Hurin principle of opposing, symmetrical complements (man–woman, sun–moon, black–white, Upper Cuzco–Lower Cuzco, the Sapa Inca–the Coya, and so forth). Anne-Marie Hocquenghem explains this division or organization of the world and its components in her work Hanan y Hurin (1984:13–17): 20

The data given by the chroniclers seem to indicate that the organization and classification of the Andean world are based on the recognition of two complementary and opposing parts, hanan and hurin ...

Each part is divided, in turn, into two halves hanan and hurin. This dualist conception generates a quadripartition that is projected into the dimension of space, time and society ... The two parts and the two halves meet and are recreated in a center that contains that which is hanan and that which is hurin in equilibrium and which is opposed to each of the points of the world where the hanan and the hurin are not balanced. This center is analogous to the world as a whole containing both hanan and hurin. (1984:13–14)

Later, she explains what belongs to each, hanan and hurin: 21 It is of hanan the right, the front, the exterior, life, disappearance, the past, the ancestor, that which collaborates. It is of hurin the left, the back, the interior, the appearance, the future, the feminine, the descendant, that which is opposed. (1984:15)

And she summarizes: 22 Hanan and hurin seem to be the two notions that allow order to be put in the world by establishing a game of correspondences and oppositions. Knowledge of the double list of what is hanan and what is hurin, of what is attracted and what is opposed, one knows and can act effectively. (1984:17)

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20 Los datos de los cronistas parecen indicar que la organización y la clasificación del mundo andino se basa en el reconocimiento de dos partes complementarias y opuestas, hanan y hurin...
Cada parte se divide a su vez en dos mitades hanan y hurin. Esta concepción doblemente dualista genera una cuadripartición que se proyecta en la dimensión del espacio, del tiempo y de la sociedad...
Las dos partes y las dos mitades se reúnen y se reproducen en un centro que contiene en equilibrio lo hanan y lo hurin y que se opone a cada uno de los puntos del mundo donde lo hanan y lo hurin no se equilibran. Este centro es análogo a la totalidad del mundo que contiene tanto hanan como hurin. (1984:13–14)

21 Es de hanan la derecha, el frente, lo exterior, la vida, la desaparición, el pasado, lo antepasado, lo que colabora. Es de hurin la izquierda, la espalda, lo interior, la aparición, el futuro, lo femenino, lo descendiente, lo que se opone. (1984:15)

22 Hanan y hurin parecen ser las dos nociones que permiten ordenar el mundo, estableciendo un juego de correspondencias y oposiciones. Cuando se conoce la doble lista de lo que es hanan y hurin, de lo que se atrae y de lo que se contraría, se sabe y puede actuar eficazmente. (1984:17)
This organization of the world into two complementary and opposed parts with a center serving as a synthesis of the two, and including both, is found in other elements of Inca culture. This notion or vision of the world is also implemented in the Inca arithmetical system by transferring the idea of complementarity and opposing elements to the two hands. Each hand, with its 5 fingers (parallel to 5 numbers), is:

... khallu (“one apart, alone, separate”) ... a prototype of the condition known as ch'ulla (“uneven, odd”; i.e., without a/its pair). Ch'ulla is a thing that has a natural pair which, for some reason, is separated from it; when the two are brought together, their unity is referred to as khalluntin, “the two separate parts of a pair, intimately bound together” ... That is, the motivation for two is the “loneliness” (ch'ulla) of one. “One” is an incomplete, alienated entity; it needs a “partner” (ch'ullantin). (Urton 1997:77–78)

Thus, going back to Hocquenghem's description, the right hand can be paralleled to hanan, and the left to hurin, and none can be considered on its own, for this would reflect a state of non-balance, of an incomplete set. Both hands bring us again to the number 10, a unit with two complementary moieties (5+5), where the 10 fingers reflect the whole. This stands in sharp contrast with the Ayamarana language, where numbers had independent names up to the number five, as the fingers of only one hand, and to Wari khipu, seemingly mostly based on this ancient Andean quinary system (see the quote from Urton 2014:217–218 above, in Section 1.3).

This intrinsic relation between the Quechua language and the Inca decimal system, as seen in numbers higher than 10, in the parallelism between the ayllu and the unit of 10, and between the inner morphology of the unit of ten and the profound hanan-hurin complementary moieties concept of the Inca makes it clear why their system could have been only decimal (in a state of khalluntin) and not quinary (in a state of ch'ulla, which would reflect an against-nature state in Inca thought).

2.3 The Censuses and the Administrative Division of the Population

The administrators of the Tawantinsuyu carried out censuses of the population at defined unknown intervals. The Inca census system was based on the division of the population into ten “calles” (Kalli = group), from 1 to 10. Both Guaman Poma de Ayala (1615:193 [195]–233 [235]) and Martín de Murúa ([1590] 1946:324–325) detail these 10 “calles,” with some minor differences in the ages given for the individuals in each group, yet following the same scheme. Murúa's detailed description of these “calles” can be summarized as follows:
<table>
<thead>
<tr>
<th>'Calles' (Kalli = groups)</th>
<th>Relative Ages</th>
<th>Description (translated from the written sources in Spanish)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>25–50; 51–65</td>
<td>'Indians for any work' (Tributaries) and 'Old people' that still knew of harvests and sowings.</td>
</tr>
<tr>
<td>3–4</td>
<td>60–end; ill, crippled</td>
<td>'Very Old' and crippled – not able to work</td>
</tr>
<tr>
<td>5</td>
<td>18–25</td>
<td>'Unmarried Youngsters' = almost tributaries, who help tributaries with their tasks: 'That they helped carry their war supplies and other things'</td>
</tr>
<tr>
<td>6–7</td>
<td>12–18; 9–12</td>
<td>'Older boys,' easy work in things pertaining to the Inca, like farming; 'Younger boys' boys evolving towards a first light work in things pertaining to the Inca: 'bird hunter.'</td>
</tr>
<tr>
<td>8–9</td>
<td>5–9; 4–5</td>
<td>Calle 8 = 'Children' playing and helping the fathers and mothers. Calle 9 = 'Younger Children' who start to walk, that is, children not fit for work for being too young.</td>
</tr>
<tr>
<td>10</td>
<td>Newborns (up to 4 years old (?))</td>
<td>'Breastfeeding babies'</td>
</tr>
</tbody>
</table>

The “calles” in this list are grouped to demarcate aptitudes for specific tasks or inability to work, with a clear division into two subgroups of 5 calles (5+5), each subgroup in turn subdivided into 2 + 2 +1 calles. In Calles 1–5 are those who can perform regular tributary tasks or help the tributaries with their tasks, or those who once worked (were once tributaries) but cannot do that anymore, while in Calles 6–10 are young children down to newborns, the first four including children that can perform light tasks and, the last one, newborns who cannot do any tasks at all, all of them not even close to being tributaries.

The tributaries, those who can work for the Tawantinsuyu in construction, in the mines, or by participating in wars, are included in the first calle, the most important for the Inca and the Tawantinsuyu. The number attached to it (1) is not to be taken literally but figuratively: this is the first, not the lowest category.

Interestingly, not only did the Inca divide the population into 10 groups for the censuses, but they also defined the population of the whole Tawantinsuyu...
as groups of 10 and powers of ten (10, 100, 1000, 10000, and so on). Furthermore, each group was administered by a Tawantsisuyu/Inca officer whose rank and name concurred with the number of people under his charge.

Father José de Acosta, among others, described this division as follows:

... Because in conquering each province, they reduced the Indians to towns and community, and they counted them by partialities, and over each ten Indians they put one that kept accounts with them, and over each hundred another, and each thousand another, and to each ten thousand another, to this one they called Huno, that was a principal position, and over all of them they put one, in each province, governor of the lineage of the Incas, to whom all obeyed and he gave account each a year of all what had happened ... ([1590] 1986:408)

Santillán ([1564] 1927:16–17, No. 10) describes the Inca administration and population with their decimal ranks:

10. He also made the said inca another division of his land to have a better account, and out of every forty thousand neighbors, he made a woman, which means province, and in each one, he put a governor who resided in it, and they called him Tocricoc, which means the one who sees it all.

... and to have more particular news of everything he made another division and gave each one hundred Indians a curaca, whom they called lord of pachacá; and among every ten curacas he chose the most skillful to command and the most man, and made curaca on the other nine, and this one was in charge of the nine curacas and his people ... they called him curaca of guaranga, which means lord of a thousand Indians.

... and for the government of an entire valley where there were many guaranges, there was a lord above all, called Huño, who was governor over the curacas of pachaca and guaranga ... And for the things related to the tribute of the inca ... was above them all the tocricoc; so that each Indian obeyed the curaca of pachaca, and that of pachaca obeyed that of guaranga and the guaranga that of Huño and all obeyed the Tocricoc.

The layout of the ‘simple knots’ representing powers of ten in the khipu (‘numbering’ from top to bottom) and its analogy to the administrative decimal division of


24 Huño means, literally, ten thousand.

25 Porque en conquistando cada provincia, luego reducían los indios, a pueblos y comunidad, y contabanlos por parcialidades, y a cada diez indios ponían uno que tuviese cuenta con ellos, y a cada ciento otro, y a cada mil otro, y a cada diez mil otro, y a éste llamaban huno, que era cargo principal. Y sobre todos éstos, en cada provincia, un gobernador del linaje de los Ingas, al cual obedecían todos y daba cuenta cada un año de todo lo sucedido...

26 To “reduce” (reducir) means gathering people from several places and putting them all in one town, thus reducing the number of small villages in an area.

27 “The one who sees it all” was called, in fact, Tukuy Rikuq and was an inspector who did not reside in the woman. The confusion between the T'uyr'ikuq = Governor and Tukuy Rikuq = Inspector is commonly found in the written sources. Nevertheless, Santillán is clearly describing here the T'uyr'ikuq or Governor, who is indeed above the Huno = the lord of 10000 households. For further research on this subject, see 2008a, 2016a.
the population in groups of powers-of-ten (‘counting’ from bottom to top), coincides with the parallelism seen previously, in the vocabularies, between the terms chunka and ayllu (an ayllu as the basic population unit = 10 households), and with the fact that the khipu, according to Guaman Poma de Ayala’s description, was the Royal Accountant’s tool, who used it to count the population and “straighten accounts with the Indians.” This “Royal Accountant of the whole empire,” according to Guaman Poma de Ayala (1615:361 [363]), was explicitly called Tauantinsuyo runaquipoc, that is “the one who annotates on the khipu the people of the four suyus.”

Thus, the decimal system also seemingly served as a base for creating an administrative system where each portion of the population was supervised by an administrator called by the decimal rank he administered: from chunka (10, parallel to an ayllu) to hunu (10 000).

Above all of these administrators was the tocricoc (t’uqrikupa), the governor of the province who was, as explained above, a khipu kamayuq of Inca blood.

2.4 Distances between two Province Capitals, Dimensions of a Province; the Tampus (Tambos) 28

The last example of how the Inca implemented the philosophy of 10 concerns the geographic structure of the Tawantinsuyu.

Two names designate administrative centers that came second to Main Cuzco in 16th–17th-centuries chronicles and vocabularies: Wamani and ‘Another/ Other Cuzco(s).” 29

Cieza de León ([1553] 1985:79–80) calls these centers “Heads of Provinces” (“Cabezas de provincias”). According to his description, these centers, serving as the provinces’ capitals, centralized their area’s tributes and comprised garrisons for warriors, a Sun Temple and Royal residences, and storehouses in great quantity. He also gives the list of localities, stating that in these localities were the “delegates of the kings”/“governors” (“delegados de los reyes’/gobernadores”), who “had great authority … and power enough to form armies and gather soldiers” (“tenían gran autoridad estos gobernadores y poder bastante para formar ejércitos y juntar gente de guerra”): “… they 30 had in all the heads of the provinces - such as Vilcas, Xauxa, Bonbon, Caxamalca, Guancabamba, Tomebamba, Latacunga, Quito, Carangui; and on the other side of Cuzco, towards midday, Hatuncolla, Ayavire, Chuquiabo, Chucuito, Paria and others that go to Chile - their delegates …” 31

28 Tampu – places along the Inca Royal Road (the Qhapaq Ñan) where travelers could eat, rest, and stay the night.
30 Referring to the Inca in Main Cuzco.
31 … tuvieron en todas las cabeceras de las provincias - como eran Vilcas, Xauxa, Bonbon, Caxamalca, Guancabamba, Tomebamba, Latacunga, Quito, Carangui; y por la otra parte del Cuzco, hacia el Mediodia, Hatuncolla, Ayavire, Chuquiabo, Chucuito, Paria y otros que van hasta Chile - sus delegados …
Cobo names these “delegates” or “governors” *tuqrikuq* ([1653] 1964:114) who are, in fact, and as described in point 1.1 above, administrators and accountants (*kipu kamayuq*) of Inca blood who studied in the Sapa Inca’s school in Main Cuzco. These governors were placed in the “heads of provinces” (province capitals).

The unit of ‘ten,’ as composed of two complementary parts of 5 each (*hanan–hurin*; a state of *khalluntin*), is implemented in the distances counted between two *wamanis* (province capital) and in the measurement of the *waman* (‘province’) itself.

There seems to be a correspondence between the distances given by the vocabularies and the chronicles between two *wamani*, between these and the *tampu*, and Guaman Poma de Ayala’s list of *tampu* on the royal road of the Inca (*qhapaq ñan*).

Pachacuti Yamqui Salcamaygua ([1613?] 1993: fol. 33v) stated that each *wamani* measured 30 leagues: ... sending the soldiers lasted three and a half months. ... And in order to do so, he ordered that in each ravine should be an *usnu*[^32] to allow seeing if they went in good military order, and in each guamanin of thirty leagues, he would count the number of people each captain has, and there he gives them food ...

However, it should be clarified here that both the chronicles and the vocabularies seem to use the terms *wamani* and *waman* as synonyms, *waman* being also a synonym for *marca* (district, province), which, in turn, is brought as a synonym for *llaqta*, both defined as city. Santo Thomas ([1560] 1951) writes:[^33]

*Marca, or Sucguamanc - province or district (fol. 150v)*

*Province, as district - marca, or suc guamanc (fol. 87)*

*District - marca (fol. 31)*

*Ciudad - llacta, or marca (fol. 39)*

Additionally, Gonçalez Holguín ([1608] 1952:175) defines both *waman* and *wamani* as “a ten-day road/journey.”[^34]

Accordingly, both *waman* and *wamani* seem to refer to the same entity and are used by the chroniclers and in the vocabularies almost indistinctively, except for Pachacuti Yamqui Salcamaygua, who calls provinces/districts *guamanin* (see above), and Santillán and Santo Thomas, who use only the term *guaman* (see above). However, Gonçalez Holguín explicitly uses both terms as synonyms meaning “a ten-day road/journey” (see above).

Gonçalez Holguín’s definition would seem to indicate that the limits of both the *waman* (province) and the location of two *wamanis* (province capitals) could be translated into a 10 walking-days distance.

[^32]: *Usnu* = a Ceremonial Plafor

[^33]: *Marca, o Sucguamanc - provincia o comarca*

*Provincia de comarca - marca, o suc guamanc*

*Comarca - marca*

*Ciudad - llacta, o marca*

[^34]: *Huc huaman o huc huamani. Camino de diez dias.*
In this same vocabulary (Gonçalez Holguín [1608] 1952: II: 550), the following translation from Spanish to Quechua supports this conclusion:

*pichqa puñuy, five days or pichqa p'unchaw purinaraqmi, five days are missing or remain to walk or *phatma waman.*

Thus, if 5 walking-days represent half the distance between two *wamans*, then a whole *waman* measures 10 walking days; this is also the distance between two administrative centers (two province capitals) or *wamani*.

Regarding the dimensions of a waman, Cieza de León ([1553] 1984:190, 205) adds that those passing through the road covered a distance of only 3 to 4 leagues a day, except for the *chasqui* (the messengers, delivery persons), who covered a larger distance. Places of rest and storehouses were placed every three to four leagues, where those making the journey could rest for the night and get supplies ([1553] 1985:313–314).

Las Casas, referring to the Royal Road ([1474–1566] 1892:164), confirms Cieza de León’s statements. He writes that between the “royal residences” (within the *Wamanis* / ‘Other Cuzcos’) were other places where they could rest, located every three or four leagues, and adds: “… this was the walking-day that the Inca King walked; and he did not want to walk anymore, so that the people serving him would not be fatigued.”

Taking into account the data presented above, the following can be concluded:

1. There were places of rest on the Royal Road, at a distance of 3–4 leagues from each other, called *tampu* in Quechua.
2. The standard distance covered in a day’s journey was about 3–4 leagues at the most.
3. A distance of 30 leagues (= one *waman*) separated between two *wamani* (province capitals).
4. Half a *waman* equals 5 walking days.
5. One *waman* or *wamani* equals 10 walking days (=30–40 leagues).
6. These 10 walking days also seem, thus, to have been the distance between two *wamani* (province capitals).

Following this logic, the distance between two *tampu* is one day’s journey or 3–4 leagues. Thus, if 10 walking days or 30 leagues separate between two *Wamani*/Other Cuzcos, there should be about 8–10 *tampu* in-between.

35 *Phatma waman* means: half a *waman*.
36 Words in bold by the author of this article.
37 *Pichca puñuy, cinco jornadas o pichca p'unchaw purinaraqmi, cinco jornadas faltan o quedan por andar o patma huaman.***
38 “… porque esta era la jornada que caminaba el Rey Inga; y no quería caminar más, porque no se fatigase la gente de su servicio
39 It should be noted that the measure of the Inca league differed from the Spanish one. The Inca league covered a greater distance than the Spanish and we cannot ascertain which league the chroniclers referred to.
As an example, and to confirm these measurements, we bring below some Royal Road sections detailed by Guaman Poma de Ayala in his chronicle. The first is the section leading from Vilcas to Main Cuzco, with its tampus. Guaman Poma de Ayala was well acquainted with the area encompassing Cuzco, Ayacucho, and Vilcashuaman and offers a detailed list of its towns, tampus, and cities. He lists the following cities and tampus in this section (1615:1090 [1100]):

**Uilcas guaman royal tambo and houses and town of uiracocha Inca**
- Laran marca, small tambo
- Oranmarca town royal tambo ...
- Andaguayllas town royal tambo
- Pingos, small tambo
- Cochacaxas royal tambo ...
- Amancay, royal tambo
- Curaguaci town royal tambo ...
- Lima royal tambo
- Xacxa uana royal tambo

**The Great City and Head of this Kingdom Santiago del Cuzco**

According to Guaman Poma’s list, 9 tampu and small tampu were situated between Vilcashuaman and Cuzco, representing a total of precisely 10 walking days (= 30 leagues), which, according to all the written sources, is the distance between two wamanis/wamanis.

The next list from Guaman Poma’s chronicle, detailing the route from Xauxa to Vilcas Waman, also shows a journey of ten days (1615:1089 [1099]–1090 [1100]):

---

40 *Uilcas guaman tanbo real y casas y pueblo de uiracocha ynga*
- Laran marca, tanbillo
- Oranmarca pueblo tanbo real...
- Andaguayllas pueblo tanbo real
- Pingos, tanbillo
- Cochacaxas tanbo real...
- Amancay, tanbo real
- Curaguaci pueblo tanbo real...
- Lima tanbo real
- Xacxa uana tanbo real

*La Gran ciudad y Cauesa deste Reyno Santiago del Cuzco*

41 Guaman Poma de Ayala wrote at the beginning of the 17th century, during the colonial period. What he described as a “small tambo” (tanbillo) could have been an important place in Inca times.

42 **XAUXA...**
- Marayblica tanbo real
- Guancayo pueblo tanbo real
- Aco tanbo real....
- Picoy tanbo real
- Parcos tanbo real
- Marcas tanbillo
- Sangaro tanbo real...
XAUCA ...
Maraybilca royal tambo
Guancayo town royal tambo
Aco royal tambo ...
Picoy royal tambo
Parcos royal tambo
Marcas small tambo
Sangaro royal tambo ...
Guamanga city and royal inn
Yllauaci small tambo
Uilcas guaman royal tambo and houses and town of uiracocha ynga

Finally, the journey from Caxamarca to Guancabamba, with 9 tampu in-between, also took exactly 10 walking days (Guaman Poma de Ayala 1615:1087 [1097]).

When comparing the names of the starting and final destinations brought forth by Guaman Poma to the lists of Wamanis/Other Cuzcos given by 16th and 17th-century chroniclers (such as Cieza de León, cited above), all these destinations are confirmed as either Wamanis or Other Cuzcos.

In conclusion, the decimal system was also implemented for measuring distances in walking-day units and for measuring the area covered by a province and half a province.

Conclusions

The Inca decimal system, although apparently similar to the Western one, was in no way parallel to it. The Inca system, counting by units of tens, was based on philosophical and structural elements inherent to Inca culture: the opposing complements hanan and hurin; paired elements (khalluntin); the ayllu, a basic unit in the Tawantinsuyu as parallel to the number 10 with ten households or interrelated elements (and not just 10 individual units as in western thinking); counting by heaps of tens transcribed to the khipu cord reflecting this new decimal thought (as compared with Wari khipus seemingly reflecting the Aymara quinary system); the 10 “calles” (kalli groups) representing different aspects of the population and used in the Inca censuses; the powers-of-ten ranks used to designate the administrators of the Tawantinsuyu; the numbers in Quechua which reflect this decimal thinking, where the basic unit is the 10, being subsequently replaced by ‘powers of ten’ (10², 10³, 10⁴) as the counting goes up; and the measuring of distances and areas in 10 or 5 walking days.

---

Guamanga ciudad y mezón real
Yllauaci tamillo
Uilcas guaman tambo real y casas y pueblo de uiracocha ynga.
Thus, we can safely ascertain that the Inca perceived their counting system, based on the unit of 10, as an inherent component of their culture and used it as a structural and philosophical model to build and administer their territory and their world.

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